

FUEL BURN OPTIMISATION OF JET-PROPELLED AIRCRAFT DURING CLIMB USING EXCEL SOLVER

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Abstract: This research paper describes the creation and usage of a fuel burn optimization program for jet-propelled aircraft using Excel Solver. The main goal is to minimize fuel usage during the climb phase, thereby reducing the operational costs and environmental impact. The study uses a mathematical model incorporating various parameters from BADA (Base of Aircraft Data) to estimate the fuel consumption. Excel Solver is then utilized to find the best flight profiles such as airspeed and rate of climb to achieve the lowest possible fuel burn. The results show a significant reduction in fuel consumption across the different flight scenarios. The program's effectiveness has been validated through various case studies presented in this paper, which demonstrates improvements in fuel burn compared to traditional flight planning methods. The findings illustrate the practical benefits of integrating optimization tools into the aviation operations, leading to cost reduction and also supporting the goal of sustainable aviation by reducing greenhouse gas emissions. Additionally, this study includes recommendations for further refining the model to enhance accuracy and applicability.

Keywords: aircraft fuel optimization; climb optimization; aircraft fuel savings; Excel solver; jet aircraft

ABBREVIATION

BADA	: Base of Aircraft Data	MTOW	: Maximum Take-Off Weight
BADA OPF	: BADA Operation Performance File	TSFC	: Thrust Specific Fuel Consumption
BADA PTF	: BADA Performance Table File		

NOMENCLATURE

ρ	: Air density (kg/m ³)	η	: Thrust specific fuel flow (kg/min(kN))
C_D	: Drag coefficient	C_{D0}	: Zero-lift drag coefficient
C_{D2}	: Lift-induced drag coefficient	C_{f1}	: First TSFC coefficient
C_{f2}	: Second TSFC coefficient	C_L	: Lift coefficient
cTc	: Thrust constants	D	: Drag force (N)
dh/dt	: Vertical speed (m/s)	f_{cr}	: Cruise fuel flow (kg/min)
f_{nom}	: Nominal fuel flow (kg/min)	g	: Gravitational acceleration (m/s ²)
H_p	: Geopotential altitude (m or ft)	$KCAS$: Calibrated air speed (knot)
m	: Aircraft mass (kg)	$T_{max\ climb}$: Maximum climb thrust (N)
T_{HR}	: Thrust acting parallel to the aircraft velocity vector (N)	T_{ratio}	: Ratio of applied thrust to maximum thrust
V_{TAS}	: True airspeed (m/s)	W	: Aircraft weight (N)

1. Introduction

In the contemporary era, the aviation sector faces a mounting pressure to enhance environmental sustainability. Climate change, which is primarily driven by the emissions of green-house gases including carbon dioxide, poses a significant global-threat. In 2016, the International Civil Aviation Organization (ICAO) implemented Carbon Offsetting and Reduction Scheme for International Aviation (CORSA) as an Annex to the Chicago Convention, which all of ICAO's member states must apply from 2019 [1]. Hence it is important to find effective solutions to address this issue and one of them is the optimization of aircraft fuel-burn.

To date, many researchers have been working on reducing the aircraft's fuel consumption during different flight phases. For instance, Mori (2020) performed optimization of the cost function using the pseudo-spectra method by varying calibrated airspeed (CAS), altitude and thrust settings [2]. The result saved 85 lbs. or 0.51% over the period of climb from 10,000 ft to 30,000 ft and cruise to reach a distance of 300 nm. In the meantime, Zhang et al. (2019) studied the continuous climb operation of A320 from sea level to FL240 with constant angle of climb [3]. The result was then compared with the traditional procedure, which showed a 12.3% reduction in fuel consumption. Furthermore, a powerful method of multi-objective environmental optimization during the departure procedure using differential evolution algorithm is presented Ref. [4]. Moreover, Wan et al. (2020) applied multi-objective optimization with genetic algorithm to the climb phase of a Boeing 737-800 aircraft [5]. It is found that, for climbing flight from an altitude of 10,000 ft to 28,000 ft and a distance of 250 km, the fuel consumption can be reduced up to 6% with the optimized aircraft speed variation. Alexandrov et al. (2022) performed optimization by varying the thrust and pitch controls to achieve fuel cost within time constraints of an aircraft during climbing from 1,500 ft to 34,000 ft and within the distance of 250 km, resulting in up to 1.5% fuel consumption reduction using the gradient-free search method [6]. However, it should be noted that all of the previously mentioned algorithms are resource-intensive and time-consuming. Hence it is taught that, with a single-objective optimization problem, a simpler method like generalized reduced gradient (GRG) method may be applicable to find solutions with less time and resources.

This research studies on optimization of fuel consumption during climb of the Airbus A320 aircraft by varying calibrated airspeed, with a given initial speed at start altitude to a given final end airspeed at the required altitude and distance. The study delves into the development of Fuel-Burn Optimization Tool of a jet-propelled aircraft during climb using GRG nonlinear method available in the Excel-Solver Add-in to solve for the optimal calibrated airspeed variation and flight trajectory.

2. Methodology

The fuel consumption model and aircraft parameters for the Airbus A320 aircraft are taken from the Base of Aircraft Data (BADA) revision 3.8, with some mathematical derivatives of the model [7]. The aircraft aerodynamics and fuel consumption models during climbing phase are described as follows. Firstly, Equation 1 shows the total energy model.

$$(T_{HR} - D)V_{TAS} = mg \frac{dh}{dt} + mV_{TAS} \frac{dV_{TAS}}{dt} \quad (1)$$

During cruise flight phase, the energy model becomes Equation 2, where drag (D), drag coefficient (C_D) and lift coefficient (C_L) are calculated using Equation 3, Equation 4 and Equation 5, respectively.

$$T_{HR} = D + m \frac{dV_{TAS}}{dt} \quad (2)$$

$$D = \frac{W \cdot C_D}{C_L} \quad (3)$$

$$C_D = C_{D_0} + C_{D_2} \times C_L^2 \quad (4)$$

$$C_L = \frac{2W}{\rho V_{TAS}^2 S} \quad (5)$$

On the other hand, during climbing and accelerated flight, the applied thrust is defined in the form of thrust ratio as given in Equation 6 and Equation 7. Meanwhile, the rate of climb can be evaluated by Equation 8. Moreover, to obtain the time spent in flight, Equation 9 can be used.

$$T_{HR} = T_{ratio} \cdot T_{max\ climb} \quad (6)$$

$$T_{max\ climb} = C_{Tc,1} \times \left(1 - \frac{H_P}{C_{Tc,2}} + C_{Tc,3} \times H_P^2\right) \quad (7)$$

$$\frac{dh}{dt} = \frac{(T_{HR} - D)V_{TAS}}{mg} - \frac{V_{TAS}dV_{TAS}}{gdt} \quad (8)$$

$$dt = \frac{w}{(T_{HR} - D)V_{TAS}} dh + \frac{w}{(T_{HR} - D)g} dV_{TAS} \quad (9)$$

In the meantime, the fuel consumption rate can be evaluated by calculating the thrust specific fuel consumption (kg/min*kN), nominal fuel flow (which is valid in all phases except for the idle descent and cruise)(kg/min) and the cruise fuel flow (kg/min) as respectively given in Equation 10, Equation 11 and Equation 12.

$$\eta = C_{f1} \times \left(1 + \frac{V_{TAS}}{C_{f2}}\right) \quad (10)$$

$$f_{nom} = \eta \times T_{HR} \quad (11)$$

$$f_{cr} = \eta \times T_{HR} \times C_{fcr} \quad (12)$$

In addition, the numerical model for estimating the acceleration of aircraft is given as Equation 13. The Trapezoid Rule [9] is applied to obtain better estimates of the time in each climb step. By integrating both sides of previous Equation 9 results in Equation 14 and Equation 15.

$$\frac{dV_{TAS}}{dt} \approx \frac{\Delta V_{TAS}}{\Delta t} \quad (13)$$

$$\int_{t_{i-1}}^{t_i} dt = \int_{h_{i-1}}^{h_i} \frac{w}{(T - D)V_{TAS}} dh + \int_{V_{i-1}}^{V_i} \frac{w}{(T - D)g} dv \quad (14)$$

$$t_i - t_{i-1} = \frac{1}{2} \left[\frac{w_i}{(T_i - D_i)V_{TAS_i}} + \frac{w_{i-1}}{(T_{i-1} - D_{i-1})V_{TAS_{i-1}}} \right] (h_i - h_{i-1}) + \frac{1}{2} \left[\frac{w_i}{(T_i - D_i)g} + \frac{w_{i-1}}{(T_{i-1} - D_{i-1})g} \right] (V_{TAS_i} - V_{TAS_{i-1}}) \quad (15)$$

In this study, three different cases have been studied. All simulated flights begin with the aircraft's MTOW of 77,000 kg, climbing from 5,000 ft to 30,000 ft under the international standard atmosphere condition and with maximum climb thrust, and cruising to reach 300 km. The three studied cases are presented as follows.

Case-1: The aircraft climbs with 200 KCAS from 5,000 ft to 30,000 ft., accelerates with level flight to 280 KCAS which is the best range airspeed at altitude of 30,000 ft ISA, and then cruises until reaching the distance of 300 km.

Case-2: The aircraft accelerates with level flight from 200 to 280 KCAS, then climbs with 280 KCAS to 30,000 ft, and then cruises at 30,000 ft until reaching the distance of 300 km

Case-3: The aircraft climbs from 5,000 ft to 30,000 ft with optimized airspeed then cruises until reaching the distance of 300 km.

For this study, the optimization statement can be written as follows:

$$\text{Minimize } \sum_{i=1}^N f_i \times (t_i - t_{i-1}) \quad (16)$$

with respect to

$$V_{TAS} \in [V_{stall}, V_{max}] \quad \text{Velocity must be within flight envelope}$$

$$T_{HR} \in [0, T_{max\ climb}] \quad \text{Thrust must not higher than maximum climb thrust}$$

and subject to

$$(t_i - t_{i-1}) > 0 \quad \text{Time spent in each segment must be positive}$$

where N represents the each segment of the altitude steps.

*Notes: While cruising, $f_i = f_{cr}$ and T_{HR} is a result from solving Equation 2, otherwise while climbing or accelerated flight, $f_i = f_{nom}$ and $T_{HR} = T_{max\ climb}$ ($T_{ratio} = 1$).

The computation process is presented as the flowchart in Figure 1. Each flight is simulated by the altitude step of 1,000 ft. In each step, the program computes the fuel flow using BADA model presented by previous Equation 1 to Equation 12. Meanwhile Equation 15 is used to compute the time. Then the total fuel consumption is calculated as accumulation of fuel mass from each step as per Equation 16. The optimization is done by varying the variable airspeed until the minimum fuel consumption is found. In this study, the Excel Solver, which is a built-in Microsoft Excel tool by Frontline Systems [8], is used for the optimization process. The Excel Solver can find an optimal (maximum or minimum) value for a formula in one cell (called the objective cell), subject to constraints or limits on the values of other formula cells on a worksheet by changing the variable cells. It has an option to solve linear, integer and non-linear problems. Due to the non-linear problem nature of the optimization in this study, the Excel Solver is set to multi-start with 1,000 starting points in hope of obtaining the global solution. In addition

to the above, a validation simulation of a flight is done using the parameters from BADA OPF and compared to the performance data from BADA PTF.

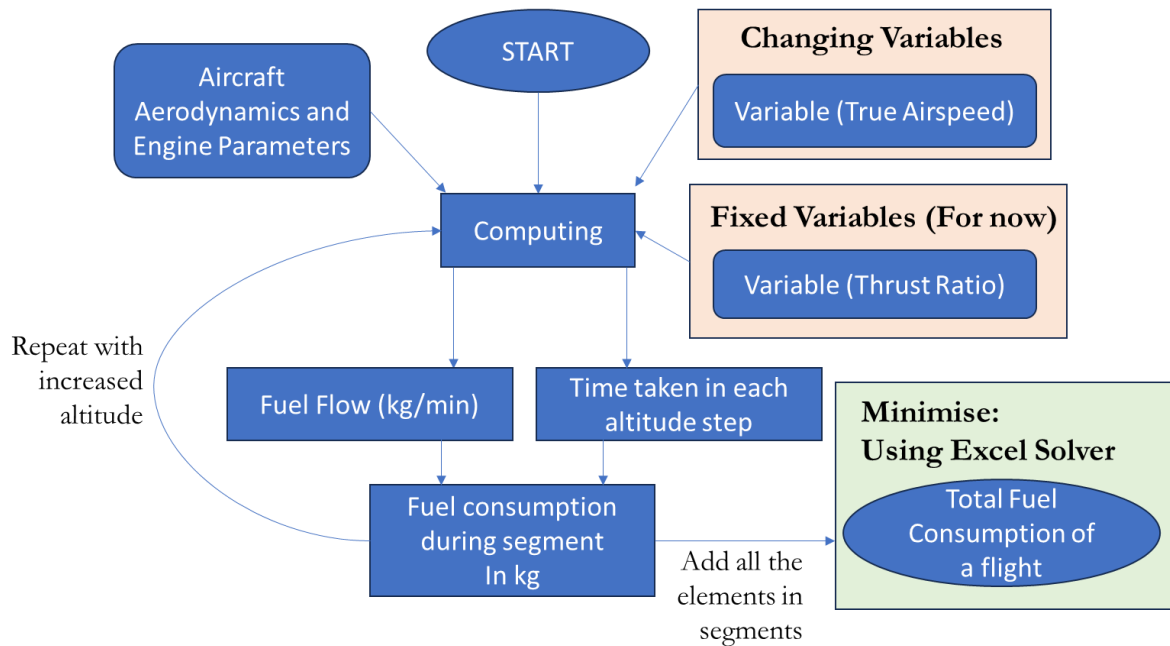


Figure 1: Flowchart describing the overall steps for this research study

In the Excel setup, each row of the spreadsheet represents the value of the respective parameters, and each column represents the simulation for each altitude ranging from 5,000 ft to 30,000 ft. This is shown in Figure 2. The steps in setting up the Excel spreadsheet are briefly described as follow:

- Step 1 :** Import aircraft’s parameters ($S, CD_0, CD_2, T_{max,0}, cTc, c_f, V_{stall}, V_{max}$) from BADA OPF (Operation Performance file) to Excel. Note that since the parameter values are specifically requested from BADA, this step is kept confidential.
- Step 2 :** Calculate the International Standard Atmosphere parameters (ρ, P) for each altitude as per BADA User Manual [7].
- Step 3 :** Calculate the remaining parameters using equations from Section 1, Section 3 and BADA User Manual [7].
- Step 4 :** For Case 1 and 2, fill the assigned airspeed of each altitude step in Figure 2 (orange fill). On the other hand, for Case-3, open and set up the Solver Parameters Dialogue as in Figure 3. When recreating, set each box to reflect your work. In Figure 2, the orange fill represents the “Changing Variable” and the summation of the values in the green cell represents the “Objective”.

Calculation							
Hp	5000	5000	6000	7000	8000	9000	10000
T	278.244	278.244	276.2628	274.2816	272.3004	270.3192	268.338
P	1760.7997	1760.799707	1695.895	1632.94173	1571.89401	1512.70762	1455.33883
rho	0.0020488	0.002048775	0.001987	0.00192745	0.0018689	0.00181171	0.00175587
P (Pa)	84307.265	84307.26454	81199.6	78185.3563	75262.3603	72428.4867	69681.6416
VCAS (knots)	200	200	200	200	200	200	200
VTAS (ft/s)	362.83584	362.8358404	368.2089	373.694138	379.294459	385.012738	390.85195
CL	0.9538331	0.953833147	0.954799	0.95524526	0.95573427	0.95626782	0.95684782
CD	0.0580273	0.058027268	0.058094	0.05812469	0.05815846	0.05819533	0.05823543
D	10327.077	10327.07651	10328.47	10323.0492	10317.6703	10312.3388	10307.0533
Tmax,climb	28512.819	28512.81859	27887.42	27266.1357	26648.9765	26035.9388	25427.0225
T/Tmax	1	1	1	1	1	1	1
T	28512.819	28512.81859	27887.42	27266.1357	26648.9765	26035.9388	25427.0225
dt new			0	27.57756	28.2029172	28.8584425	29.5618022
V TAS (kt)		214.9756134	218.1591	221.409016	224.727135	228.115143	231.574801
eta		0.791785799	0.794133	0.79652868	0.79897488	0.80147261	0.80402317
fuel flow		95525.14456	98511.22	96607.1598	94710.4646	92820.9982	90938.6234
fuel	0	0	45.27832	45.4100622	45.553275	45.7325997	45.9514869
fue acc	0	0	45.27832	90.6883811	136.241656	181.974256	227.925743
time acc	0	0	27.57756	55.7804788	84.6389213	114.200723	144.518856
d acc (km)	0	0	3.117613	6.35355769	9.71448232	13.2093776	16.8482074

Figure 2: Calculation table for Step 2 and Step 3 (taken partially from 5,000 ft to 10,000 ft)

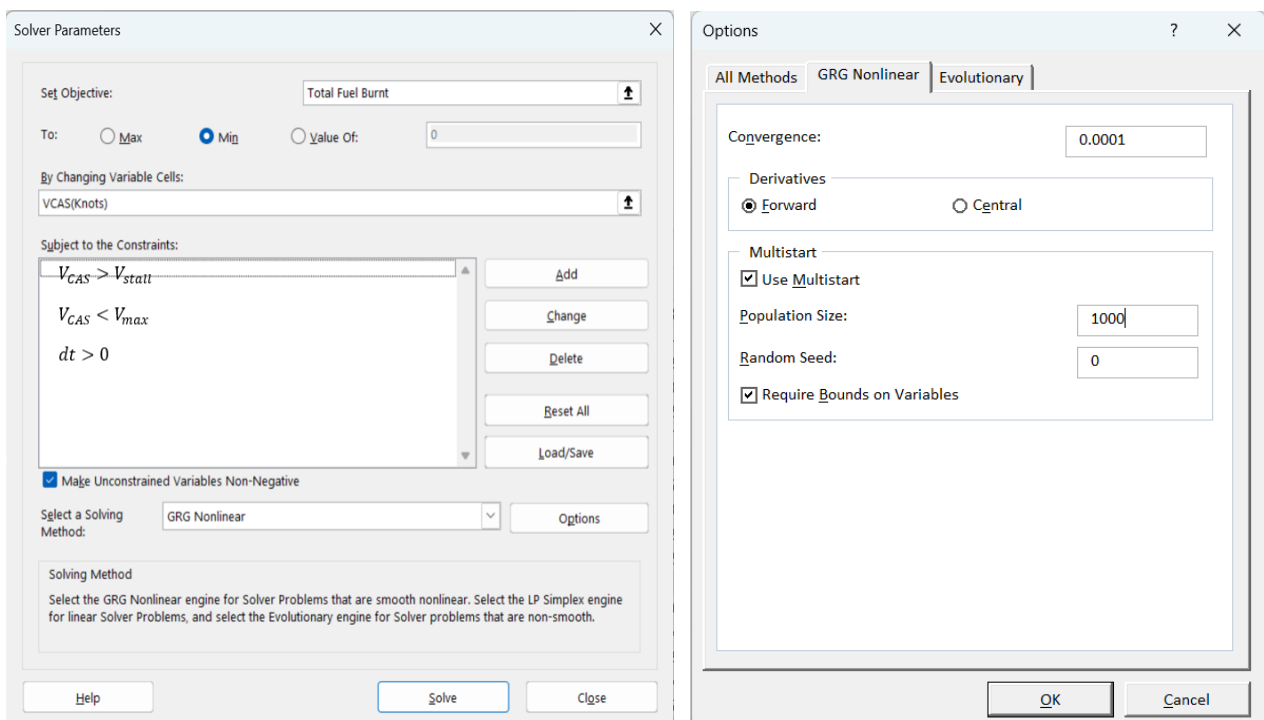


Figure 3: Solver parameters dialogue set-up

3. Results and Discussion

In the first part of the study, the performance model is validated with the performance data from BADA PTF (BADA PTG dated 01 Apr 2010). The comparison of the results is shown in Figure 4. As

can be observed, the result using the present model agrees well with the reference model with a mean absolute error (MAE) of 0.0377 kg/min or a mean absolute percentage error (MAPE) of 0.02%.

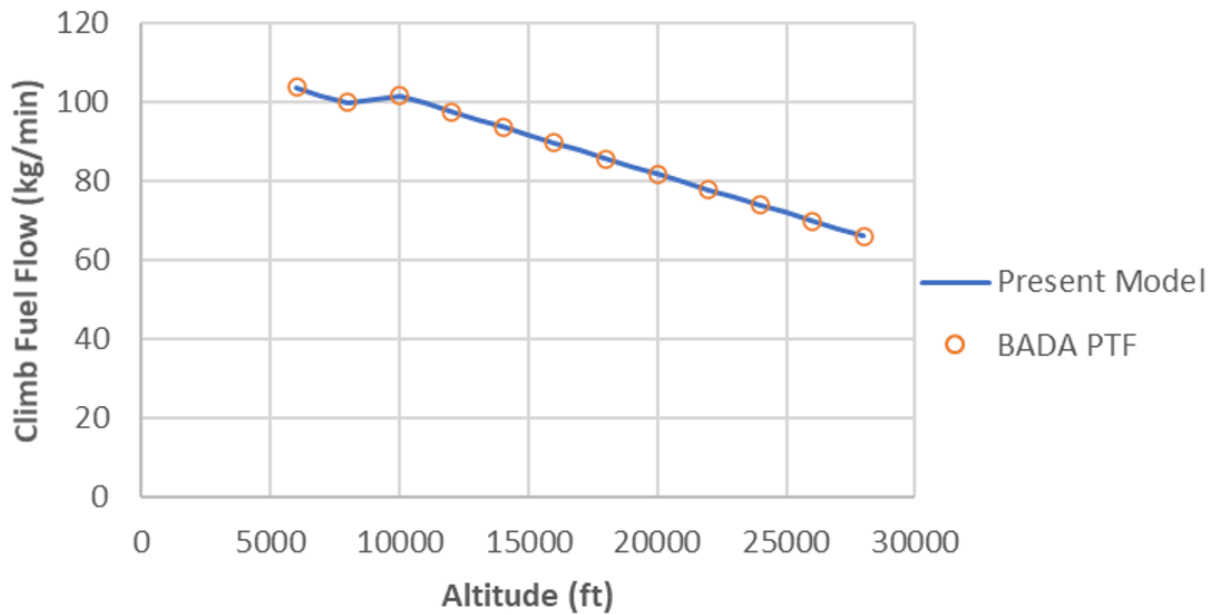


Figure 4: Validation of climb fuel flow

In the simulation of Case-1, the flight begins with climbing from 5,000 ft at constant KCAS of 200 knots up to 30,000 ft, then accelerating with level flight to 280 KCAS, and cruising until reaching the distance of 300 km. Computation of aerodynamics-based BADA model then provides the true airspeed, rate of climb, duration to climb and cruise, distance and the amount of fuel used until completing the flight. The true airspeed profile and altitude flight trajectory are illustrated in Figure 5. The calculation provides the total fuel burn of 1915.3 kg.

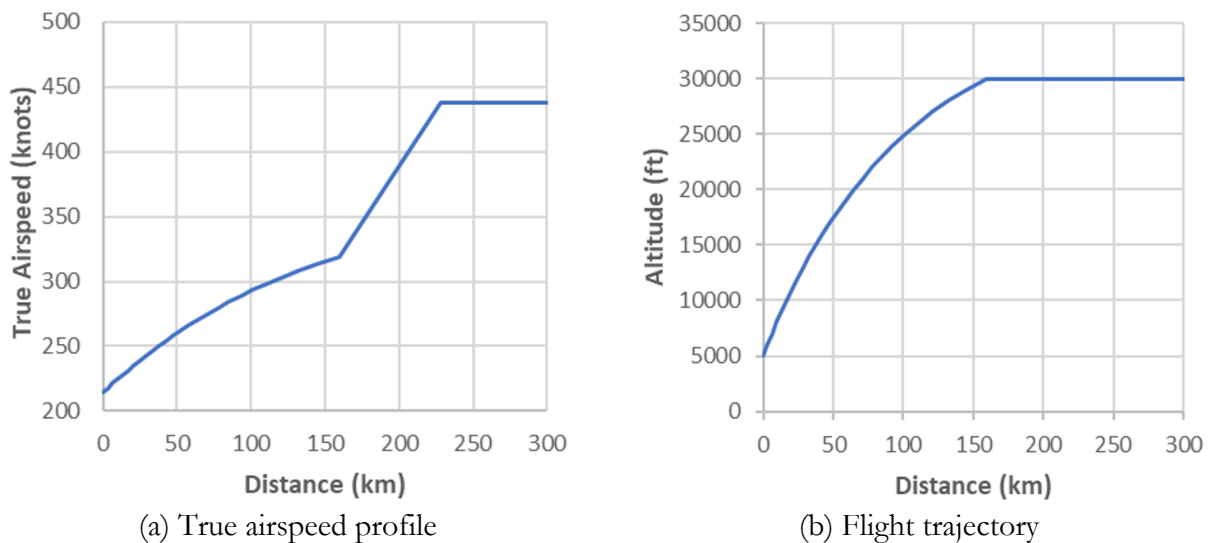


Figure 5: True airspeed profile and flight trajectory of Case-1

In the simulation of Case-2, the flight begins with accelerating from CAS of 200 knots to 280 knots at an altitude of 5,000 ft, then climbing to 30,000 ft with constant CAS of 280 knots, and cruising until

reaching the distance of 300 km. The computation provides the true airspeed profile and flight trajectory as shown in Figure 6. The calculation provides the results of total fuel burn as 1699.5 kg.

Last but not the least, in the simulation of Case-3, the flight begins with accelerating from CAS of 200 knots to the optimized airspeed at altitude of 5,000 ft, then climbing to 30,000 ft with the optimized airspeed, and cruising until reaching the distance of 300 km. The optimized speed is determined using the Excel Solver. The computation provides the true airspeed profile and flight trajectory as shown in Figure 7. The calculation provides the results of total fuel burn as 1689.3 kg.

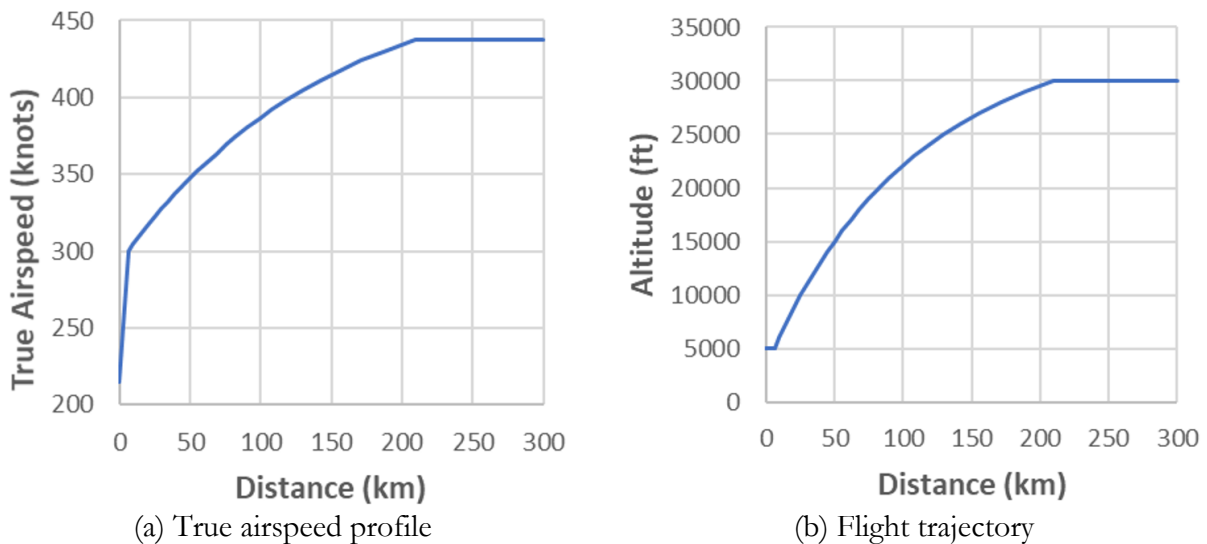


Figure 6: True airspeed profile and flight trajectory of Case-2

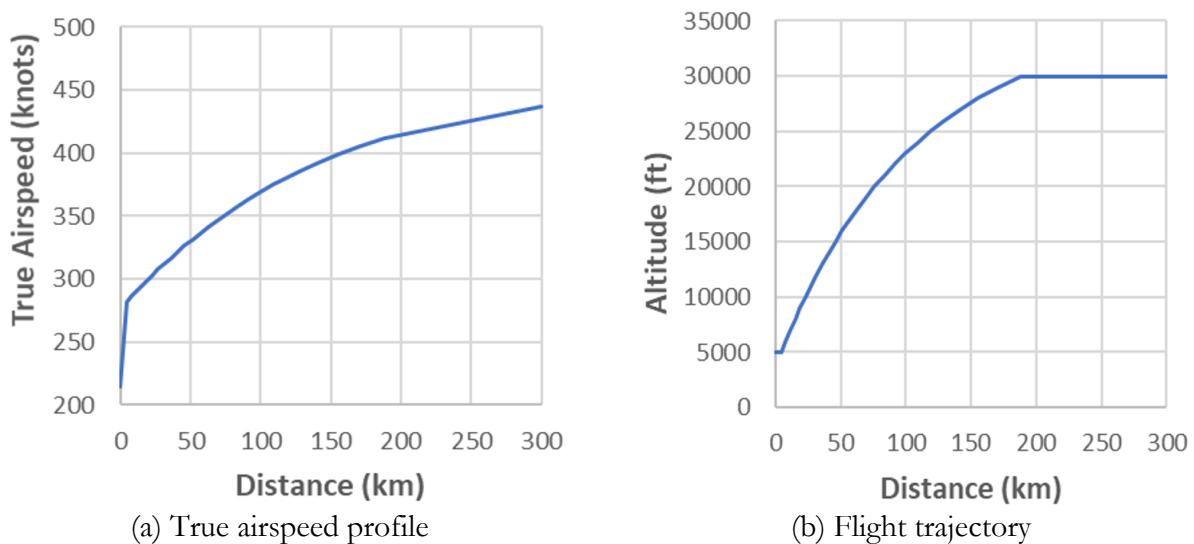


Figure 7: True airspeed profile and flight trajectory of Case-3

From the case studies mentioned above, the optimization of airspeed (Case-3) has been shown to help reduce the fuel burn by 226 kg when compared to Case-1 and 10.2 kg when compared to Case-2. Nevertheless, the amount saved from Case-2 is minimal due to climb calibrated airspeeds are constant and similar to Case-2 during the climb phase. Figure 8 compares the trajectory of the different cases.

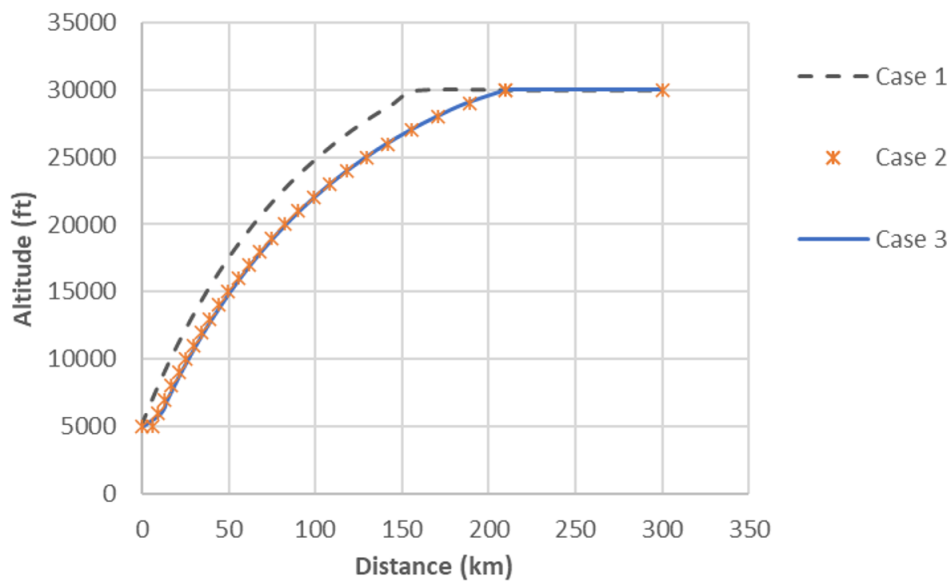


Figure 8: Flight trajectories of different cases

4. Conclusion

In this study, a fuel burn optimization program has been created and used for jet-propelled aircraft using Excel Solver. The objective is to minimize the fuel usage during the climb phase, thereby reducing the operational costs and also environmental impact. Based on obtained results, a significant reduction in fuel consumption across the different flight scenarios has been demonstrated using the optimization program. Nevertheless, there are some limitations in this study that may need to be addressed in further research. Firstly, the model has not yet involved the flight envelope into consideration. Secondly, the model utilizes the International Standard Atmospheric conditions. Further upgrade to the model can be achieved to accommodate non-standard conditions. Finally, further research on optimized variable airspeed and thrust during climbing and accelerating should be considered.

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